Type of Material Failure

Deformation failure: is a change in the physical dimensions or shape of a component that is sufficient for its function to be lost.

Fracture: cracking to the extent that a component is separated into pieces.

Corrosion: the loss of material due to chemical action.

Wear: surface removal due to abrasion or sticking between solid surfaces that touch.

(if caused by gas or liquid, it is called erosion).

Deformation
  - elastic
  - plastic

Fracture
  - Static loading
    - Overload fracture
    - Creep
  - Cyclic loading: fatigue
Elastic deformation

\[ \sigma = \frac{F}{A_0} \]

Engineering stress: \( \sigma = \frac{F}{A_0} \)

Engineering strain: \( \epsilon = \frac{L - L_0}{L_0} = \frac{\Delta L}{L_0} \)

\[ \sigma = E \epsilon \]

\( \sigma \) vs. \( \epsilon \) graph:

- Elastic deformation is fully recoverable upon unloading.
2. Plastic deformation

Yielding

* Yielding is determined as the initial departure from linearity of the stress-strain curve.

- A convention is established to measure yielding stress $G_Y$
  - a) A straight line is constructed parallel to the elastic portion of the $G$-$E$ curve at some specific strain offset, usually 0.002.
  - b) The stress corresponding to the intersection of the line and the $G$-$E$ curve is defined as yield strength $G_Y$.

Elastic recovery after Yielding

The unloading curve is parallel to the initial elastic portion of the curve. The strain regained during unloading corresponds to the strain recovery.

* Plastic deformation is non-recoverable upon unloading.

(Permanant change in shape) — Referring to slides 2
Ductile vs. Brittle fracture

\[ G \] Ductile

\[ F \] Brittle

* Ultimate strength \( G_u \): the highest stress reached before fracture.

* Strain of fracture \( G_f \): it measures ductility, otherwise called percent elongation.

Materials having high \( G_u \), high \( G_f \) are called/said to be tough.

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referring to slides 3-6
True stress and strain

True stress is defined as the load $F$ divided by instantaneous area $A_i$:

$$\sigma_T = \frac{F}{A_i} \leftarrow \text{instantaneous area}$$

Assume no volume change during deformation,

$$A_i L_i = A_0 L_0$$

$$\Rightarrow \text{True stress:} \quad \sigma_T = \frac{F}{A_i} = \frac{F L_i}{A_0 L_0} = \frac{F}{A_0} \cdot \frac{L_0 + \Delta L}{L_0} = \sigma (1 + \varepsilon)$$

True strain:

$$\epsilon_T = \int_0^{L_i} \frac{dL}{L} = \ln \left( \frac{L_i}{L_0} \right) = \ln \left( \frac{L_0 + \Delta L}{L_0} \right)$$

$$\Rightarrow \epsilon_T = \ln (1 + \varepsilon)$$

* It equation is only valid to the onset of necking.
Revisit: Engineering stress: \( \sigma = \frac{F}{A_0} \) \( \Rightarrow \) applied force \( \Rightarrow \) initial cross-sectional area

Engineering strain: \( \varepsilon = \frac{\Delta L}{L_0} \) \( \Rightarrow \) elongation \( \Rightarrow \) initial length

Elastic deformation:

- Elastic deformation is fully recoverable upon unloading.
- Most engineering materials require some time for complete recovery.

*onelasticity is time-dependent elastic behavior.
Yield stress

intersection between the drawn line and the $\sigma$-\(\varepsilon\) curve.

Elastic recovery and Strain hardening

Ultimate strength \(\sigma_u\)
Fracture strain \(\varepsilon_f\)

Accumulated \(\varepsilon_p\)
Recovered \(\varepsilon_e\)

\(\varepsilon-\sigma\) curve for brittle materials
True stress:

\[ \sigma_T = \sigma (1 + \varepsilon) \]

accounting for area change

True strain:

\[ \varepsilon_T = \ln (1 + \varepsilon) \]

\[ \varepsilon_T < \varepsilon \]

![True Stress-Strain and Engineering Stress-Strain diagram](image)
Fatigue and cyclic loading

* Fatigue occurs at a stress level below $\sigma_f$
* Fatigue is catastrophic, occurring by initiation and propagation of cracks.
* Fracture surface is usually perpendicular to the direction of an applied tensile stress.

Cyclic stress

![Graph of cyclic stress](image)

Mean stress:

$$\sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}$$

Stress amplitude:

$$\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}$$

S-N curve

Stress versus the logarithm of the number $N$ of cycles to failure.

![Graph of S-N curve](image)

* Fatigue limit: the S-N curve becomes horizontal at a high $N$ value
fatigue strength: the stress at which failure will occur for some specific number of cycles. (e.g. $10^8$)

fatigue life: the number of cycles to cause failure at a specified stress level.

Three steps leading to fatigue failure:

1) crack initiation — nucleates from surface scratch, surface step, dent

2) crack propagation — characterized by two type of markings termed benchmarks and striations. (clam shell marks)

3) final failure — occurs very rapidly once the advancing crack has reached a critical size.
Factors Affecting Fatigue Life

1) Mean Stress \( G_m \uparrow \), Stress Amplitude \( \uparrow \), Fatigue Life \( \downarrow \)

\[
\begin{array}{c}
6_{m3} > 6_{m2} > 6_{m1} \\
6_{m1} \\
6_{m2} \\
6_{m3}
\end{array}
\]

Cycles to failure, \( N \)

2) Surface effects:
   - Surface polishing enhances fatigue life
   - Increasing surface hardness.

Thermal fatigue is normally induced at elevated temperature by fluctuating thermal stress. Thermal stress \( \sigma \) depends on the coefficient of thermal expansion \( \alpha \), the modulus of elasticity \( E \) and temperature change \( \Delta T \) according to

\[ \sigma = \alpha E \Delta T \]
Creep

* Creep is deformation that accumulates with time.
* Occurs in materials at elevated temperatures when they are exposed to static mechanical stress.

> metals: $T > T_m$ where $T_m$ is melting temperature.

Creep behavior

* A typical creep test consists of subjecting a specimen to a constant load or stress while maintaining the temperature constant.
* Deformation or strain is measured and plotted as a function of elapsed time.
* If the tests control constant-load, they yield information of an engineering nature.

Constant-stress tests provide a better understanding of the mechanisms of creep.
Creep curve consists of three regions.

I: **Primary creep** or **transient creep** occurs first.

Creep rate (the slope of the curve) continuously decreases.

Material is experiencing an increase in creep resistance or strain hardening.
Deformation becomes more difficult as the material is strained.

II: **Secondary creep** or **steady-state creep**

The creep rate is constant — the curve becomes linear.

\\(<\text{slowest})\\

The competing processes of strain hardening and **recovery** reach a balance.

**Recovery** is the process by which a material becomes softer and retains its ability to experience deformation.
Tertiary Creep

Creep rate is accelerated, leading to failure. The creep failure is frequently termed *rupture*.

A neck may form at some point of deformation region, then results in a decrease in the effective cross-sectional area and increase in strain rate.

**Stress and temperature effects**

With either increasing stress or temperature,

1. the instantaneous strain at the time of stress application increases

2. the steady-state creep rate increases

3. the rupture lifetime decreases.
Relationship between stead-state creep rate and stress

\[ \dot{\varepsilon}_s = k_1 \sigma^n \]

creep rate, material constants

applied stress

\[ \Rightarrow \ln \dot{\varepsilon}_s = n \ln \sigma + \ln k_1 \]

\[ \ln \dot{\varepsilon}_s \text{ versus } \ln \sigma \text{ yields a straight line with slope of } n. \]

Creep rate and temperature

diffusion is governed by an Arrhenius equation:

\[ D = D_0 \exp\left(-\frac{Q}{RT}\right) \]

Boltzmann's constant

\[ \Rightarrow \dot{\varepsilon}_s = k_2 \exp\left(-\frac{Q}{RT}\right) \]

Temperature

\[ \ln \dot{\varepsilon}_s = \ln k_2 - \frac{Q}{RT} \]

\[ \ln \dot{\varepsilon}_s \text{ versus } \ln \sigma \text{ yields a straight line with slope of } n. \]

\[ \ln \dot{\varepsilon}_s \text{ versus } \frac{1}{T} \text{ yields a straight line with slope of } -\frac{Q}{R}. \]
Creep rate equation, including temperature, and stress

\[ \dot{\varepsilon}_t = A \sigma^n \exp\left(-\frac{Q}{RT}\right) \]

where \( n \) is stress exponent, depending on material

\( Q \) is activation energy,

\[ \Rightarrow \ln \dot{\varepsilon}_t = \ln(A \sigma^n \exp(-Q/RT)) \]

\[ \Rightarrow \ln \dot{\varepsilon}_t = \ln A + n \ln \sigma - \frac{Q}{RT} \]
e.g. $\sigma \rightarrow \delta$

The steady state creep rates for aluminum at 260°C were measured at two different states:

<table>
<thead>
<tr>
<th>Creep rate $\dot{\varepsilon}$ ($\text{m}^3$)</th>
<th>Stress $\sigma$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.0 \times 10^{-4}$</td>
<td>3</td>
</tr>
<tr>
<td>$3.65$</td>
<td>25</td>
</tr>
</tbody>
</table>

Compute $\dot{\varepsilon}$ at a stress of 10 MPa and 260°C.

Solution: temperature is constant,

$$\ln \dot{\varepsilon}_s = \left( \ln A - \frac{Q}{kT} \right) + n \ln \sigma$$

$$\Rightarrow \begin{cases} 
\ln(2.0 \times 10^{-4}) = a + n \ln 3 \quad \text{(1)} \\
\ln(3.65) = a + n \ln 25 \quad \text{(2)}
\end{cases}$$

$$2-0 \Rightarrow n = \frac{\ln(3.65) - \ln(2.0 \times 10^{-4})}{\ln 25 - \ln 3} = 4.63$$

$$4.63 \rightarrow 0 \Rightarrow a = \ln(2.0 \times 10^{-4}) - 4.63 \ln 3$$

$$4.63 \times 10^{-4} = 5.3 \times 10^{-2} \text{ h}^{-1}$$
Summary:

- Engineering stress, strain \( \Rightarrow \) True stress, strain.
  - Initial/undeformed configuration \( \Rightarrow \) Instantaneous configuration.

- Elastic deformation, onelasticity.
  - Time independent \( \Rightarrow \) Time dependent elastic behavior.

- Yield stress, ultimate strength, strain hardening.
  - Plastic deformation induced strengthening.

- Ductile fracture and brittle fracture.
  - Cup-and-cone profile \( \Rightarrow \) Relatively flat.

- Fatigue, cyclic stresses, the S-N curve.
  - Crack initiation, propagation, final fracture.

- Creep, stress and temperature effects.
  - Primary, secondary, tertiary.

\[ \dot{\varepsilon}_S = A \sigma^n \exp\left(-\frac{Q}{RT}\right) \]
Basic Crystal Structures

Three common crystal structures: BCC, FCC, HCP

BCC → body-centered cubic
FCC → face-centered cubic
HCP → hexagonal close-packed

Elastic Deformation: is associated with stretching, but not breaking, the chemical bonds between the atoms in a solid.

\[
\text{Stress: } \sigma = \frac{P}{A} \quad \text{cross-sectional area of material per atom}
\]

\[
\text{Strain: } \varepsilon = \frac{x - x_e}{x_e}
\]

\[
\Rightarrow \text{modulus } E = \left. \frac{d\sigma}{d\varepsilon} \right|_{x=x_e} = \frac{x_e}{A} \left. \frac{dP}{dx} \right|_{x=x_e}
\]

\(E\) relates to bond energy, interatomic spacing, nature of the bond.
Consider two planes of atoms being forced to move slowly past one another (See slide).

Shear stress versus displacement \( \alpha \), in a sinusoidal variation

\[
T = T_b \sin \frac{2\pi \alpha}{b}
\]

where \( T_b \) is maximum value as \( T \) varies with \( \alpha \); hence, theoretical shear strength.

Shear strain: \( \gamma = \frac{\alpha}{h} \)

\[
\Rightarrow \text{Shear modulus } G = \left. \frac{dT}{dr} \right|_{\alpha=0} = h \left. \frac{dT}{d\alpha} \right|_{\alpha=0}
\]

Obtaining \( \frac{dT}{dr} \) from Eq. (1) and substituting its value at \( \alpha=0 \) gives \( T_b \):

\[
T_b = \frac{Gb}{2\pi h}
\]
The ratio $b/h$ varies with the crystal structure and is generally around 0.5 to 1, so the estimation of $T_b$ is on the order of $G/10$.

We can also estimate theoretical tensile strength

$$G_b = E/10.$$
Elastic Deformation and Elastic Modulus

At low stress level, the stress and strain are proportional to each other through the relationship

\[ \sigma = E \varepsilon. \]

This is known as Hooke's law, and \( E \) is the modulus of elasticity or Young's modulus.

The modulus measures a material's resistance to elastic deformation.

\[ p \xlongequal{\text{equilibrium distance}} x_e \]

Stress: \( \sigma = \frac{P}{A} \), strain: \( \varepsilon = \frac{x - x_e}{x_e} \)

\[ E = \frac{d \varepsilon}{d x} \bigg|_{x_e} = \frac{x_e}{A} \frac{d P}{d x} \bigg|_{x=x_e} \text{ bond strength.} \]
Shear stress varies with displacement $x$ in a sinusoidal form:

$$\tau = \tau_b \sin \frac{2\pi x}{b}$$

where $\tau_b$ is the theoretical shear strength.

Shear modulus $G = \frac{d\tau}{dx} \bigg|_{x=0}$, where shear strain $\gamma = \frac{x}{h}$

$$\Rightarrow G = h \frac{d\tau}{dx} \bigg|_{x=0}$$

$$\frac{d\tau}{dx} = \tau_b \cos \frac{2\pi x}{b} \cdot \frac{2\pi}{b} = \frac{2\pi \tau_b}{b} \cos \frac{2\pi x}{b}$$

$$\left. \frac{d\tau}{dx} \right|_{x=0} = \frac{2\pi \tau_b}{b}$$

$$2) + 3) \Rightarrow G = \frac{2\pi h}{b} \tau_b$$

$$\Rightarrow \tau_b = \frac{Gb}{2\pi h} \approx \frac{G}{10}$$
Dislocation and Plastic Deformation

* The discrepancy between theoretical strength and actually measured value can be explained by dislocation.

* Dislocation is a one-dimensional defect around which some of the atoms are misaligned.

* Two types of dislocation: edge dislocation, screw dislocation.

* Burgers vector, denoted by \( \mathbf{b} \), is the magnitude and direction of the lattice distortion associated with a dislocation.

* Plastic deformation corresponds to the motion of large number of dislocation.
Plastic strain and dislocation movement

Consider a crystal volume $h/l$ containing edge dislocations. Applying shear stress, the dislocation will glide.

If a dislocation moves completely across the slip plane through the distance $d$, it contributes $b$ to the total displacement $D$.

If it moves a distance $x_i$, the contribution is the fraction $(x_i/d)$ of $b$, i.e. $\frac{x_i}{d}b$.

If $N$ dislocations move, the total displacement is

$$D = \frac{b}{d} \sum_{i=1}^{N} x_i$$

So the macroscopic plastic shear strain $\varepsilon$ is given by

$$\varepsilon = \frac{D}{h} = \frac{b}{hd} \sum_{i=1}^{N} x_i$$
Let \( \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \), be average distance moved by a dislocation.

\[ \Rightarrow \quad \varepsilon = \frac{N b}{h d} \bar{x} \]

With dislocation density \( \rho = \frac{N l}{h d} = \frac{N}{h d} \)

\[ \Rightarrow \quad \varepsilon = b \rho \bar{x} \]

The plastic strain rate

\[ \dot{\varepsilon} = \frac{d \varepsilon}{d t} = b \dot{\rho} \bar{x} + b \rho \bar{v} \], \( \bar{v} \) is dislocation velocity.

if \( \rho \) is constant.

\[ \dot{\varepsilon} = b \rho \bar{v} \]